Investigations in Number, Data, and Space® is a K–5 mathematics program with four major goals:

- to offer students meaningful mathematical problems
- to emphasize depth in mathematical thinking rather than superficial exposure to a series of fragmented topics
- to communicate mathematics content and pedagogy to teachers
- to substantially expand the pool of mathematically literate students

Program

The Investigations program is based on years of research about how children learn mathematics. Each grade level consists of a set of separate units, each offering 2–8 weeks of work. These units of study are presented through investigations that involve students in the exploration of major mathematical ideas, and may revolve around related areas, for example, addition and subtraction or geometry and fractions. The students may have a Student Activity Booklet or they may have Student Sheets, Homework Sheets, Game Instructions, Practice Pages, and Assessment Tasks to show and share what they know and are able to do.

Benefits for Students

Approaching the mathematics content through investigations helps students develop flexibility and confidence in approaching problems, fluency in using mathematical skills and tools to solve problems, and proficiency in evaluating their solutions. Computational fluency of basic number facts develops through understanding of number relationships and practice in games and activities. Students also use many ways to communicate about their mathematical thinking and their multiple strategies to solve problems while their enjoyment and application of mathematics grows.

The investigations are carefully designed to invite all students into mathematics—girls and boys; members of diverse cultural, ethnic, and language groups; and students with different strengths and interests. The context of the problems often call on students to share experiences from their family, culture, or community.

The curriculum eliminates barriers—such as work in isolation from peers, or emphasis only on speed and memorization—that exclude some students from participating successfully in mathematics.

All Students Learn and Participate

The following aspects of the curriculum ensure that all students are included in significant mathematical learning:

- Students spend time exploring problems in depth
- They find more than one solution to many of the problems they work on
- They develop their own strategies and approaches based on their knowledge and understanding of mathematical relationships
- They choose from a variety of concrete materials and appropriate technology, including calculators, as a natural part of their everyday mathematical work
- They express their mathematical thinking through drawing, writing, and talking
- They work in a variety of groupings—as a whole class, individually, in pairs, and in small groups
- They move around the classroom as they explore the mathematics in their environment and talk with their peers

Introducing Investigations to Families
Things to Observe
in an Investigations Classroom

- Students are actively involved in all aspects of the lesson
- Students use concrete mathematical manipulative objects as a natural part of instruction
- Students solve problems as their primary purpose in math class
- Students work fewer computational exercises and more problems that require higher-order thinking
- Students use calculators as a natural part of doing mathematics
- Students engage in activities that help develop spatial and number sense
- Students are asked to justify and explain their thinking
- Students work on extended problems
- Students participate in group learning experiences
- Students participate in data collection and analysis
- Students are encouraged to use computers when needed
- Students take home Family Letters regularly
- Students are assessed with formal and informal assessment
- Students record and reflect in math journals
- Students’ estimation skills are incorporated naturally
- Students reinforce mental mathematics daily
Assessment plays a critical role in teaching and learning, and it is an integral part of the *Investigations* curriculum. Assessment is an ongoing process. Teachers observe students’ discussions and explanations of their strategies on a daily basis and examine their work as it evolves. While students are busy recording and representing their work, working on projects, sharing with partners, and playing mathematical games, teachers have many opportunities to observe their mathematical thinking.

Assessment activities may involve writing and reflecting, a discussion or brief interaction between student and teacher, or the creation and explanation of a product. In most cases, the assessments require that students show what they did, write or talk about it, or do both.

End-of-Unit Assessments are designed to assess students’ understanding of the most important mathematical ideas of a unit through their solution of problems. Checklists of Mathematical Emphases are the most important ideas and processes students encounter in an investigation. They are the students’ learning objectives.
Family Letters
in Investigations in Number, Data, and Space

Just as it is important for children to read at home, it is essential for them to do math at home and to sometimes involve family. Family Letters are informative letters for each unit that introduce families to the mathematics their children are doing in each unit. The letters will convey the idea that for the students the need to understand how to figure out and how to solve an actual problem using multiple approaches, strategies, and processes is the essence of the mathematics in *Investigations in Number, Data, and Space*.

Each letter gives families an idea of what their children are doing in class, explains how this is connected to the rest of their children’s lives, describes some of the homework children will be doing, and provides ideas for families about how to expand on the mathematics that is being done at school.

Dear Family,

During the next few weeks, your child will be working on a mathematics unit about multiplication and division, called *Things That Come in Groups*.

Your child will be making lists of items that come grouped in different amounts—things like 2 shoes in a pair, 7 days in a week, 12 eggs in a carton. Later the class will use these lists to write their own story problems.

Students will also work with the 100 chart, which shows the numbers from 1 through 100, organized in a systematic 10-by-10 grid. On this chart, your child will discover patterns in the multiples of a given number. We will also make arrays, or objects arranged in rows and columns to form rectangles of different shapes and dimensions. For example, an array with the dimensions 2 x 3 would look like

with a total of 6 squares. Students play games with Array Cards, learning to recognize the dimensions and the total number of small squares in each rectangle. The aim is a meaningful introduction to multiplication that helps students visualize what multiplication combinations “look” like.

Through our activities, students will be learning many multiplication pairs or “facts.” Students will naturally learn many of these multiples through repeated use. They will practice counting by different multiples—for example, by 3’s (3, 6, 9, 12, …) as one way of learning multiplication relationships. Students will be encouraged to visualize what these “facts” mean and what their relationships are—for example, that 6 x 4 is four more than 5 x 4 or that 4 x 8 is double 2 x 8.

The emphasis of this unit is on understanding what multiplication and division mean. The children will be asked to make sense of different multiplication and division situations. They will develop their own ways for thinking about and writing about these. Family members can help with many of the assignments during this unit. For example, you can help your child look for things that come in equal groups. You can take turns skip counting on the 100 chart with your child. And you can play the number games that your child brings home.

Sincerely,
Homework
in Investigations in Number, Data, and Space

In *Investigations*, homework is a vehicle for connecting school mathematics with students’ everyday lives. Homework is an extension of classroom work. Students are asked to work on problems that extend and solidify their mathematical understanding. Sometimes homework offers review and practice of work done in class, sometimes preparation for upcoming activities, and sometimes numerical practice that revisits work from earlier units. Homework may include:

- Number games that may be played more than once to give students multiple opportunities to clarify mathematical ideas and reinforce basic facts
- Problems to solve that ask students to use their prior knowledge of math skills and ideas
- Tasks such as collecting data or taking measurements to be used solving a problem in class
- Facts to practice to become competent in estimating, and accurate and fluent in computation

Homework plays a role both in supporting students’ learning and in helping inform families about the ways in which students work with mathematical ideas in *Investigations in Number, Data, and Space*. 
Games are used throughout the *Investigations* program as a way to engage students in important mathematical ideas. The game format is one that students enjoy, so the potential for repeated experiences with a concept or skill is great. Because most games involve at least one other player, students are likely to learn strategies from each other whether they are playing cooperatively or competitively.

The more times students play a mathematical game, the more opportunities they have to practice important skills and to think and reason mathematically. The first or second time that students play, they focus on learning the rules. Once they have mastered the rules, their real work with the mathematical content begins.

Students need many opportunities to play mathematical games, not just during math time, but at other times as well. Games played as homework can be a wonderful way of communicating with parents.

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**Playing Games**

The Importance of Playing Games More Than Once

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In the *Investigations* program students need to take an active role in mathematics. They must do more than get the correct answers; they must think critically about their ideas, give reasons for their answers, and communicate their ideas.

We can help students develop their thinking and reasoning by asking questions such as:

- Does this remind you of other problems you’ve worked?
- What have you come up with so far?
- Where do you think you should start?
- What is the problem asking you to do?
- Would drawing a picture help?
- How can I help you?
- How did you find your answer?
- Why does that work?
- Is there another way?
- How do you know it solves the problem?

Over time, students become more comfortable and confident thinking about their solution, recording it, and explaining it to others.
Investigations uses children’s literature to support the mathematical ideas the students investigate. Below are some suggestions to look for at the local public library.

### Literature in Investigations in Number, Data, and Space

- **K–1**
  - Allen, Pamela
    - *Who Sank the Boat?*
  - Burns, Marilyn
    - *The Greedy Triangle*
  - Crews, Donald
    - *Ten Black Dots*
  - Henkes, Kevin
    - *Chrysanthemum*
  - Grossman, Virginia
    - *Ten Little Rabbits*
  - Lankford, Mary
    - *Hopscotch Around the World*
  - Reid, Margarette
    - *The Button Box*
  - Sturges, Philemon
    - *Ten Flashing Fireflies*

- **2–3**
  - Carle, Eric
    - *The Very Hungry Caterpillar*
  - Dee, Ruby
    - *Two Ways to Count to Ten*
  - Emberley, Ed
    - *Ed Emberley's Picture Pie*
  - Giganti, Paul, and Donald Crews
    - *Each Orange Had 8 Slices*
  - Hong, Lily Toy
    - *Two of Everything*
  - Hutchins, Pat
    - *The Doorbell Rang*
  - Merriam, Eve
    - *12 Ways to Get to 11*
  - Pinczes, Elinor J
    - *One Hundred Hungry Ants*

- **4–5**
  - Anno, Mitsumasa
    - *Anno’s Magic Seeds*
  - Birch, David
    - *The King’s Chessboard*
  - Clement, Rod
    - *Counting on Frank*
  - Leedy, Loreen
    - *Fraction Action*
  - Mathis, Sharon Bell
    - *The Hundred Penny Box*
  - Pittman, Helena Clare
    - *A Grain of Rice*
  - Schwartz, David
    - *If You Made a Million*
  - Tompert, Ann
    - *Grandfather Tang’s Story*
The *Investigations* curriculum incorporates the use of two forms of technology in the classroom: calculators and computers. Calculators are assumed to be standard classroom materials, available for student use in any unit. Just as with other tools, students must learn both how to use calculators correctly and when they are appropriate to use. This knowledge is crucial for daily life, as calculators are now a standard way of handling numerical operations, both at work and at home.

Computers are explicitly linked to one or more units at each grade level; they are used with a unit on 2-D geometry at each grade, as well as with some of the units on measuring, data, and changes. Students can use computers to approach and visualize mathematical situations in new ways. The computer allows students to construct and manipulate geometric shapes, see objects move according to rules they specify, and turn, flip, and repeat a pattern.
# Try-Me Activities

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<td>Recognize, name, know the value, count coins</td>
<td>Count things around the home past 10, identify numbers on signs and in an elevator, set the table for your family size</td>
<td>Learn single-digit facts, starting with doubles $1 + 1, 2 + 2, 3 + 3$, $5 - 5, 4 - 4, 3 - 3$</td>
<td>Name days of week, months of year, seasons</td>
<td>Compare items by length, weight, mass</td>
<td>Count and compare: cars vs trucks; record and compare: sunny/cloudy days</td>
<td>Name shapes (circle, square, rectangle) and find representations in the home and outside</td>
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<td>Make change, make 25 and 50 cents in multiple ways, manage an allowance</td>
<td>Count by 2, 5, 10, count past 100, look for patterns, practice skip counting</td>
<td>Know addition and subtraction facts to 20, begin to learn multiplication and division fact families ($3 \times 4 = 12, 4 \times 3 = 12, 12 \div 3 = 4, 12 \div 4 = 3$)</td>
<td>Know minutes/hour; hours/day; read an analog clock</td>
<td>Use measurement tools: ruler, tape measure, scale, etc.</td>
<td>Survey and graph family members’ “favorites”, describe the data, and ask “why” and “how” questions</td>
<td>Name and describe differences of 2-D and 3-D objects in the home; explore area and volume of cereal boxes</td>
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<td>Help grocery shop, compare prices, budget allowance for saving/spending short/long term</td>
<td>Practice skip counting – starting at 2, 3, 7, 9; count past 1000 counting by 25, 50, 250</td>
<td>Know multiplication and division facts to 12, fluently and accurately</td>
<td>Practice scheduling, time management; determine elapsed time</td>
<td>Participate in cooking, building, measuring, arranging bedroom</td>
<td>Discuss newspaper graphs or charts, discuss the probability of likely and unlikely events</td>
<td>Draw up a proposal for new carpet and paint for your bedroom; determine cost</td>
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